

# MULTIPLICITY DISTRIBUTIONS IN QCD AT VERY HIGH ENERGIES

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## Abstract

It is shown that QCD is able to predict very tiny features of multiplicity distributions at very high energies which demonstrate that the negative binomial distribution (and, more generally speaking, any infinitely divisible distribution) is inappropriate for precise description of experimental data. New precise fits of high energy multiplicity distributions can be derived.

In this report, I briefly review the results of several cited below papers in which it has been shown that it is possible to solve the strongly non-linear integro-differential equations of QCD for the generating function of multiplicity distributions. From the solutions obtained, it follows the prediction about quite peculiar behavior of cumulants of the distribution which was never found before. The accelerator data support the prediction. Thus, more precise fits can be proposed to use in the cosmic ray range of very high energies.

First, let us introduce some definitions related to the normalized multiplicity distribution

$$P_n = \sigma_n / \sum_{n=0}^{\infty} \sigma_n \quad (1)$$

of probabilities  $P_n$  for  $n$ -particle (prong) events. Its generating function  $G(z)$  is defined as

$$G(z) = \sum_{n=0}^{\infty} (1+z)^n P_n, \quad (2)$$

factorial moments are

$$F_q = \frac{\sum n(n-1)\dots(n-q+1)P_n}{(\sum nP_n)^q} = \frac{1}{\langle n \rangle^q} \frac{d^q G(z)}{dz^q} \Big|_{z=0} \quad (3)$$

and cumulants are

$$K_q = \frac{1}{\langle n \rangle^q} \frac{d^q \log G(z)}{dz^q} \Big|_{z=0}. \quad (4)$$

We use also their ratio

$$H_q = K_q / F_q. \quad (5)$$

For the sake of simplicity, we consider here only a single gluon jet with the total energy in its c.m.s. equal to  $Q$  and introduce the evolution parameter as  $y = \ln(Q^2/Q_0^2)$ , where  $Q_0^2 = \text{const.}$  The generating function satisfies the QCD equation [1,2]

$$G'(y) = \int_0^1 dx \left( \frac{1}{x} - \Phi_r(x) \right) \gamma_0^2 [G(y + \ln x) G(y + \ln(1-x)) - G(y)], \quad (6)$$

where  $\Phi_r(x) = (1-x)(2-x(1-x))$  is the regular part of the Altarelli-Parisi kernel and  $\gamma_0^2 = 6\alpha_S/\pi$  ( $\alpha_S$  is QCD coupling constant). It is the integro-differential non-linear equation with shifted arguments in non-linear part and, therefore, it seems impossible to find its solution. However, in a series of papers [3-7] it was shown that such a solution exists in higher-order perturbative QCD for the running coupling constant. Moreover, in the case of the fixed coupling constant, one was able to get an exact solution of the equation [8,9] (both in gluodynamics and in QCD with quarks and gluons). The decisive role in that progress is played by the usage of formulae (3),(4) proposed in [4] and by notion of the well-known relation between factorial moments and cumulants:

$$F_q = \sum_{m=0}^{q-1} C_m^{q-1} K_{q-m} F_m, \quad (7)$$

where  $C_m^{q-1}$  are the binomial coefficients. Usage of (3),(4),(6) enables one to get additional relation between  $F_q$  and  $K_q$  which together with (7) provides the knowledge of any function  $F_q, K_q, H_q$  and, therefore, of multiplicity distribution  $P_n$ . I shall not delve into mathematical details leaving room for physics discussion. One can learn them from cited papers [3-10].

MAIN PHYSICS CONCLUSIONS ARE:

1. KNO-scaling is valid at very high energies.
2. The shape of the distribution is much narrower than in the lowest perturbative approximation (called DLA).
3. The tail of the distribution falls off approximately as  $\exp[-a(n/\langle n \rangle)^\mu]$  with  $\mu > 1$ .
4. The cumulants  $K_q$  acquire negative values while factorial moments are always larger than 1.
5. Factorial moments are steadily increasing with their rank  $q$  while cumulants oscillate with increasing amplitude at very large  $q$ . (Integer values of  $q$  are considered. The extension to non-integer values was proposed in [11].)

6. The ratio  $H_q$  also oscillates with first minimum positioned at  $q = 5$  that reveals a new expansion parameter of QCD.

7. This parameter shows that one should be careful in application of perturbative QCD to multiparticle production.

8. The property 6) has been confronted to experiment and has found very good support of it (see [12]).

9. The properties 4), 6) demonstrate that the negative binomial distribution (so popular nowadays) is inappropriate for precise fits of multiplicity distributions because its cumulants are always positive.

10. Moreover, all infinitely divisible distributions are inappropriate since they have positive cumulants.

11. The cluster models with the Poissonian distribution of clusters are inappropriate too.

The implications of these conclusions are to be studied yet. However, it is remarkable that perturbative QCD becomes a powerful tool in describing the soft processes when properly treated with higher-order terms taken into account. Thus, the widely spread opinion that perturbative QCD is inapplicable to soft processes should be reconsidered. The only (however, important) shortcoming is the hadronization stage treated still either in the framework of the local parton-hadron duality hypothesis or in Monte-Carlo simulations with various assumptions. However, from above consideration it is clear that qualitative features are well reproduced (and even predicted!) by QCD at the partonic level. The demonstrated in [3-10] possibility of the proper treatment of the partonic stage provides some hope. In particular, the energy dependence of partonic multiplicities [10] and the ratio of the partonic multiplicities in gluon and quark jets [8, 10] give good chances for further studies of the hadronization stage when compared with experimental data.

What concerns cosmic ray studies, it is not clear yet how the above findings influence the fragmentation region multiplicities that is of utmost importance for these investigations. It should be analyzed in combination with the knowledge of rapidity spectra in hadron-hadron and hadron-nucleus collisions.

In conclusion, the non-linearity of QCD has important implications for soft hadronic processes and it can be accurately treated to provide new predictions for very high energy particle interactions.

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